Important Formulas

Electrical & Electronics Measurements

POWERED BY:

gradeup

Chapter 1
Characteristics of Instruments and Measurement Systems

Measurements

Measurement is a process by which one can convert physical parameters to meaningful numbers. The measuring process is one in which the property of an object or system under consideration is compared to an accepted standard unit, a standard defined for that particular property.

Static Characteristics

1. Accuracy
It is the closeness with which an instrument reading approaches the true value of the quantity being measured.
2. Precision
It is a measure of the reproducibility of the measurements. It is a measure of degree of agreement within a group of measurements.

Remember :

- Precision is not the guarantee of accuracy.
- An instrument with more significant figure has more precision.

3. Sensitivity
It is the ratio of the magnitude of output signal to the magnitude of input signal applied to the instrument.

$$\text{Sensitivity} = \frac{\text{Output}}{\text{Input}}$$

Note : ……………………………………………………………

- An instrument requires high degree of sensitivity.
- $\text{Sensitivity} \propto \frac{1}{\text{Deflection factor}}$

4. Resolution

The smallest change in input which can be detected with certainity by an instrument is its resolution.

5. Linearity

The output is linearly proportional to the input. For a linear instrument the sensitivity is constant for the entire range of instrument. Linearity is the most important parameter compared to all other parameters.

Remember : …………………………………………………..

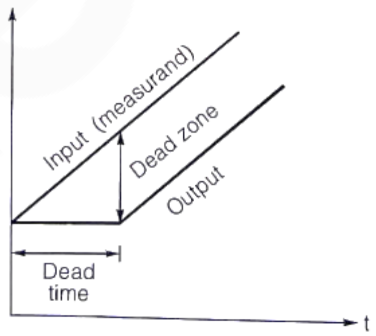
- Linearity is more important than the sensitivity.
- Accuracy is more important than resolution.

6. Dead Zone

It is the largest change of input quantity for which there is no output of the instrument.

7. Dead time

Time required by an instrument to begin to respond to the change in a measurand.



8. Range and Span
The difference between the maximum and minimum values of the scale is called range. The maximum value of the scale is called span.

Errors

Error = Measured value – True value

Error = Accuracy

- Static Error

$$\delta A = A_m - A_t$$

where,

A_m = Measured value of quantity or Actual value

A_t = True value of quantity or Nominal value

Relative static error

$$\epsilon_r = \frac{\delta A}{A_t}$$

Static correction

$$\delta C = A_t - A_m = -\delta A$$

Static sensitivity

$$\text{Static sensitivity} = \frac{\Delta q_0}{\Delta q_i}$$

Where,

Δq_0 = Infinitesimal change in output

Δq_i = Infinitesimal change in input

Non-linearity (N.L.)

$$\text{N.L.} = \frac{\left(\begin{array}{l}\text{Max. deviation of output from} \\ \text{the idealized straight line}\end{array}\right)}{\text{Full scale deflection}} \times 100$$

Error at desired value =

$$\frac{\text{Full scale value} \times \text{Error at full scale}}{\text{Desired value}}$$

Combination of Quantities with Limiting Errors

Sum or Difference of Two or More than Two Quantities

Let $X = \pm X_1 \pm X_2 \pm X_3 \pm X_4$

Where

$\pm \delta X_1$ = Relative increment in quantity X_1

$\pm \delta X_2$ = Relative increment in quantity X_2

$\pm \delta X$ = Relative increment in X

$\frac{\delta X_1}{X_1}$ = Relative limiting error in quantity X_1

$\frac{\delta X_2}{X_2}$ = Relative limiting error in quantity X_2

$\frac{\delta X}{X}$ = Relative limiting error in X

Product or Quotient of Two or More than two Quantities

Let $X = x_1\ x_2\ x_3$ or $X = \frac{x_1}{x_2 x_3}$ or

$$X = \frac{1}{x_1 x_2 x_3}$$

$$\frac{\delta X}{X} = \pm\left(\frac{\delta x_1}{x_1} + \frac{\delta x_2}{x_2} + \frac{\delta x_3}{x_3}\right)$$

Composite Factors

Let $X = x_1^n . x_2^m$

$$\frac{\delta X}{X} = \pm\left(n\frac{\delta x_1}{x_1} + m\frac{\delta x_2}{x_2}\right)$$

Arithmetic Mean

$$\overline{X} = \frac{\Sigma x}{n} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

where, $x_1, x_2, \ldots, x_n$ = Readings or samples

n = Number of readings

Deviation

$$d_n = x_n - \overline{X}$$

Note : ……………………………………………………………………

Algebraic sum of deviation is zero.

……………………………………………………………………………

Average deviation

$$\overline{D} = \frac{\Sigma |d|}{n} = \frac{|d_1| + |d_2| + \ldots |d_n|}{n}$$

Standard deviation

For n > 20

$$\text{S.D.} = \sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{d_1^2 + d_2^2 + \ldots + d_n^2}{n}}$$

For n < 20

$$\text{S.D.} = s = \sqrt{\frac{\Sigma d^2}{n-1}}$$

Variance

For n > 20

$$V = \sigma^2 = \frac{\Sigma d^2}{n}$$

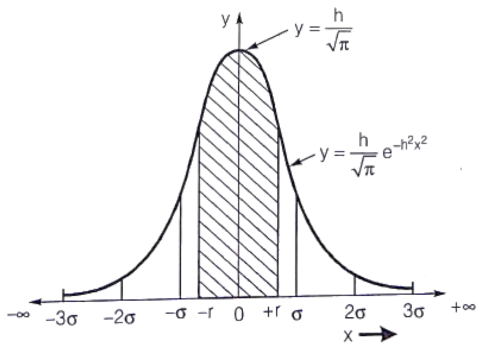
For n < 20

$$V = s^2 = \frac{\Sigma d^2}{n-1}$$

Normal or Gausian Curve of Errors

1. For Infinite Numbers of Reading

$$y = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-x^2 / 2\sigma^2\right)$$



Where,

x = magnitude of deviation from mean,

y = number of readings at any deviation x, (the probability of occurance of deviation x)

σ = standard deviation

Precision Index

$$h = \frac{1}{\sigma\sqrt{2}}$$

Probable error (P.E.)

$$r = \frac{0.4769}{h}$$

Average deviation

$$D = \frac{r}{0.8453} = \frac{1}{\pi h^2}$$

Standard deviation

$$\sigma = \frac{r}{0.6745} = \frac{1}{h\sqrt{2}}$$

$$P.E. = r = 0.8453\,\bar{D} = 0.6745\,\sigma$$

2. For Finite Numbers of Reading

For n > 20

$$P.E. = r = 0.6745\sqrt{\frac{\Sigma|d|^2}{n}}$$

For n < 20

$$P.E. = r = 0.6745\sqrt{\frac{\Sigma|d|^2}{n-1}}$$

Standard deviation of mean

$$\sigma_m = \frac{\sigma}{\sqrt{n}}$$

Standard deviation of standard deviation

$$\sigma_\sigma = \frac{\sigma_m}{\sqrt{2}}$$

Variance of combination of components

Let X = f(x_1, x_2, …., n)

$$V_x = \left(\frac{\partial X}{\partial x_1}\right)^2 V_{x_1} + \left(\frac{\partial X}{\partial x_2}\right)^2 V_{x_2} + \dots + \left(\frac{\partial X}{\partial x_n}\right)^2 V_{x_n}$$

Where,

$V_{x_1}, V_{x_2}, \dots V_{x_b}$ = Variance of $x_1, x_2, \dots x_n$

Standard Deviation of Combination of Components

Let X = f(X_1, X_2, ……., X_n)

$$\sigma_x = \left(\frac{\partial X}{\partial x_1}\right)^2 \sigma_{x_1}^2 + \left(\frac{\partial X}{\partial x_2}\right)^2 \sigma_{x_2}^2 + \dots + \left(\frac{\partial X}{\partial x_n}\right)^2 \sigma_{x_n}^2$$

Where, $\sigma_{X_1}, \sigma_{X_2}, \dots, \sigma_{X_n}$ = Standard deviation of x_1, x_2, …., x_n

Probable Error of Combination of Components

Let X = f(X_1, X_2, ……., X_n)

$$r_x = \sqrt{\left(\frac{\partial X}{\partial x_1}\right)^2 r_{x_1}^2 + \left(\frac{\partial X}{\partial x_2}\right)^2 r_{x_2}^2 + \dots + \left(\frac{\partial X}{\partial x_n}\right)^2 r_{x_n}^2}$$

Where, $r_{X_1}, r_{X_2}, \dots r_{X_n}$ = Probable error of x_1, x_2, …., x_n

Uncertainty of Combination of Components

Let X = f(X_1, X_2, ……., X_n)

$$w_x = \sqrt{\left(\frac{\partial X}{\partial x_1}\right)^2 w_{x_1}^2 + \left(\frac{\partial X}{\partial x_2}\right)^2 w_{x_2}^2 + \dots + \left(\frac{\partial X}{\partial x_n}\right)^2 w_{x_n}^2}$$

Where, $w_{X_1}, w_{X_2}, \dots w_{X_n}$ = Uncertainties of x_1, x_2, …., x_n

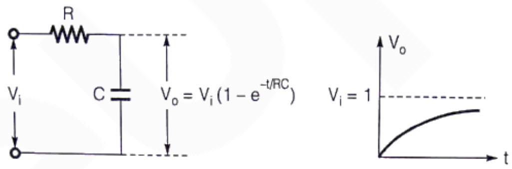
Order of Instrument

1. Zero Order System

As input changes, output also changes immediately called zero order system. Example : Resistor.

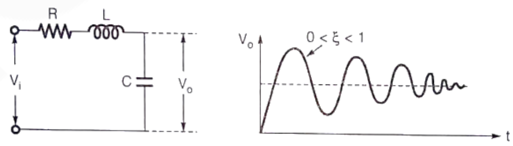
2. First Order System

As input changes, output also changes but not immediately, it takes some delay but without oscillation. Example : heater.



3. Second Order system

As input changes, output also changes, with some delay and oscillation.



Remember : ………………………………………………………..

The analog instruments are of second order instrument which has damping factor (ξ) between 0.6 to 0.8. It is an underdamped system.

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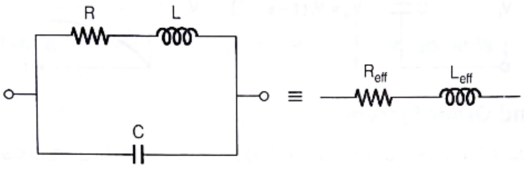
Standards

Quantity	Unit	Definition
Length	Metre	The length of path travelled by light in an interval of $\frac{1}{299792458}$ sec.
Time	Second	9.192631770 × 10^9 cycles of radiation from vapourised cesium-133 atom.
Temp.	Kelvin	The temperature difference between the absolute and the triple point of water is defined as 273.16ºK.
Voltage	Volt	Standard cell voltage of Weston cell i..e 1.0186 V.
Current	Ampere	One ampere is the current flowing through two infinite long parallel conductor of negligible cross section placed 1 meter apart produced a force of 2 × 10^{-7} N/m.

Chapter 2
Circuit Components
(Resistors, Inductors, Capacitors)

Frequency Errors in Resistors



(Equivalent circuit of a resistor at low and medium frequencies)

Effective resistance

$$R_{eff} = \frac{R}{1+\omega^2 C\left(CR^2 - 2L\right)}$$

Effective inductance or residual inductance

$$L_{eff} = \frac{L - CR^2}{1+\omega^2 C\left(CR^2 - 2L\right)}$$

$$\tan\phi = \frac{X_{eff}}{R_{eff}} = \frac{\omega L_{eff}}{R_{eff}} = \frac{\omega\left(L - CR^2\right)}{R} = \omega\left(\frac{L}{R} - CR\right)$$

Where, ϕ = Phase deflection angle

Time constant

Condition for resistance to remain independent of frequency

$$CR^2 = 2L$$

Condition for resistance to show no inductive effect

$$CR^2 = L$$

Effective resistance for zero effective inductance

$$R_{eff} = \frac{R}{1 - \omega^2 LC}$$

Quality factor

$$Q = \frac{\omega L}{R}$$

Frequency Errors in Inductors

Effective resistance

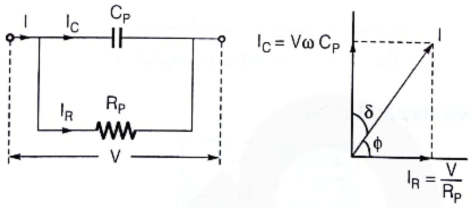
$$R_{eff} \frac{R}{\left(1 - \omega^2 LC\right)^2}$$

Effective inductance

$$L_{eff} = L\left(1 + \omega^2 LC\right)$$

Capacitor

1. Parallel Representation



Dielectric loss

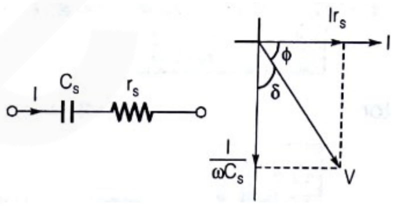
$$P_L = \omega C_p V^2 \tan\delta$$

Dissipation factor

$$D = \tan\,\delta = \frac{1}{\omega C_p R_P}$$

Where, δ = loss angle of the capacitor.

2. Series Representation



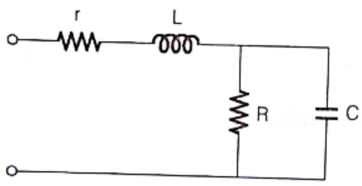
Dielectric loss

$$P_L = \frac{I^2}{\omega C_s}\tan\delta$$

Dissipation factor

$$D = \tan\,\delta = \omega\, C_s\, r_s$$

Frequency Errors in Capacitors



Equivalent Circuit of a Capacitor

Effective capacitance

$$C_{eff} = \frac{C}{1 - \omega^2 LC}$$

1. **For Medium Frequency**

Effective capacitance

$$C_{eff} = C\left(1 + \omega^2 LC\right)$$

Effective series resistance

$$R_{eff} = r + \frac{R}{1 + \omega^2 R^2 C^2}$$

Where, r = resistance of load

Loss angle

$$\tan\delta = \frac{1-\omega^2 LC}{\omega r + \frac{1}{\omega CR}}$$

2. For Law Frequency

Effective capacitance

$$C_{eff} = C + \frac{1}{\omega^2 CR^2}$$

Effective series resistance

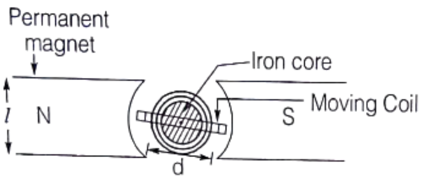
$$R_{eff} = \frac{R}{1+\omega^2 C^2 R^2}$$

Loss angle

$$\tan\delta = \omega Cr + \frac{1}{\omega CR}$$

Chapter 3
Galvanometers

D'Arsonval Galvanometer



Deflecting torque

$$T_d = BiNA = Gi$$

Where,

B = Flux density in air gap; Wb/m^2

i = Current through moving coil; A

N = Number of turns in coil

A = ld = Area of coil; m^2

l, d = Length of vertical and horizontal side (width) of coil

A = ld = Area of coil; m2

l, d = Length of vertical and horizontal side (width) of coil respectively; m

G = Displacement constant of galvanometer

Controlling torque

$$T_c = K\theta_f$$

Where,

K = Spring constant of suspension; Nm/rad

θ_f = Final steady deflection of moving coil; rad

Final steady deflection

$$\theta_f = \left(\frac{NBA}{K}\right)i = \left(\frac{G}{K}\right)i$$

Dynamic behaviour of Galvonometers

Torques in Galvonometers

Inertia torque

$$T_j = J\frac{d^2\theta}{dt^2}$$

Where,

J = moment of inertia of moving system about the axis of rotation; kg-m^2,

θ = deflection at any time t; rad.

Damping torque

$$T_D = D\frac{d\theta}{dt}$$

Where, D = damping constant

Controlling torque

$$T_C = K\,\theta$$

Where, K = control constant

Deflecting Torque

$$T_d = Gi$$

Equation of motion

$$T_j + T_D + T_C = T_d$$

$$J\frac{d^2\theta}{dt^2} + \frac{Dd\theta}{dt} + K\theta = Gi$$

Note : ....

If D^2 < 4 kJ, galvanometer is under-damped.

If D^2 = 4 kJ, galvanometer is critically damped.

If D^2 > 4 kJ, galvanometer is over-damped.

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Total resistance of galvanometer circuit for critical damping

$$R = \frac{G^2}{2\sqrt{KJ}}$$

External series resistance required for critical damping

$$R_e = R - R_g = \frac{G^2}{2\sqrt{KJ}} - R_g$$

Where, R_g = Resistance of galvanometer

Sensitivity

Current sensitivity

$$S_i = \frac{\theta_f}{i} = \frac{G}{K}\ \text{rad/A}$$

$$S_i = \frac{d}{i \times 10^6} \text{ scale divisions/μA}$$

$$S_i = \frac{2000\,G}{K \times 10^6} \text{ mm/μA}$$

Voltage sensitivity

$$S_v = \frac{d}{iR_g \times 10^6} \text{ scale division/μV}$$

Megohm sensitivity

$$S_o = \frac{d}{i \times 10^{-6}} \text{ MΩ/scale division}$$

Remember : …………………………………………………………

Sensitive galvanometer is one which produces a large deflection for a small current.

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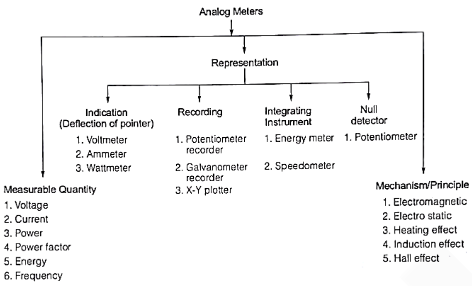
Chapter 4
Analog Meters

Classification of Analog Meters



Torque in Analog Meter

1. Deflecting Torque (T_D)

   Deflecting torque is proportional to quantity under measurement. This torque deflect the pointer away from initial or zero position.

   $$T_D \propto \text{Measurable quantity}$$

2. Controlling Torque (T_C)

   The controlling torque is opposite to deflecting torque. When, deflecting torque equals to controlling torque, pointer comes to final steady state position.

   At equilibrium, $T_C = T_D$

Note : ………………………………………………………………

Control torque is also used to bring the pointer in zero initial position, if there is no deflecting torque.

Except in PMMC, in all other instruments if the control spring is failed or broken then pointer moves to the maximum position of scale.

Control torque is provided by

(i) Spring control

(ii) Gravity control

………………………………………………………………………………

3. **Damping Torque**

   It is used to damp out oscillation at final steady state position. The time response of the instrument depends on damping torque.

   Damping torque provided by :

   (i) Air friction damping : Used where low magnetic fields are produced

   (ii) Fluid friction damping : Used where deflecting torque is minimum.

   (iii) Eddy current damping : Used where permanent magnet produces the required deflecting torque.

Error in Analog Meters

1. **Frictional Error**

   To reduce the frictional error, the torque to weight ratio of the instrument should be high.

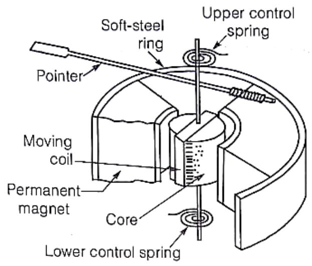
2. **Temperature Error**

   Due to change in temperature, change in resistance of meters and shunts and series multiplier occurs. To reduce this effect, resistances are made up of manganin material.

3. **Frequency Error**

   Due to change in frequency, error produce in instrument because change in frequency cause change in reactance. To reduce this error, a capacitance is used in case of voltmeter and for ammeter, the time constant and shunt impedances are maintained at same value.

Permanent Magnet Moving Coil (PMMC)



Deflection torque

$$T_D = nBAI$$
$$T_D = GI$$

Where,

G = nBA

n = Number of turns

B = Flux density

A = Area of core

I = Current to be measured

Final steady state deflection

$$\theta = \left(\frac{G}{K}\right)I$$

Where, K = Spring constant

Note : ………………………………………………………………..

- PMMC instrument measures only DC or average values.
- Scale is linear.
- Spring is used for controlling torque.
- Damping torque provided by eddy current damping.
- It has more, torque to weight ratio so accuracy and sensitivity is higher compare to other instrument.
- In direct measurement, the PMMC measures upto a current of 50 mA or a voltage of 100 mV, without any external device.

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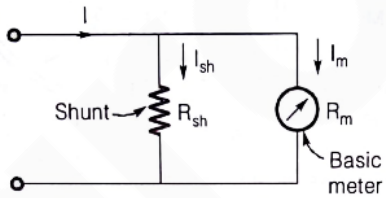
Enhancement of Ammeters and Voltmeters

1. Ammeter Shunts



Basic ammeter circuit

$$I_{sh} R_{sh} = I_m R_m$$

$$I = \left(1 + \frac{R_m}{R_{sh}}\right) I_m$$

Where,

I = Current to be measured;

I_m = I_{fs} = Full scale deflection current; A

R_m = Internal resistance of meter; Ω

R_{sh} = Resistance of the shunt; Ω

Shunt resistance

$$R_{sh} = \frac{R_m}{m-1}$$

Where, $m = \frac{I}{I_m} = 1 + \frac{R_m}{R_{sh}}$

m = Multiplying factor for shunt

Note : ………………………………………………………………..

To reduce the temperature effect, swamp resistance made up of manganin is added in series with ammeter.

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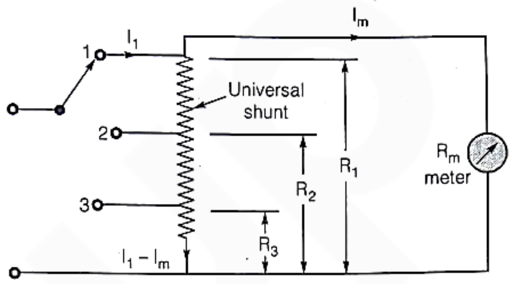
2. Universal or Ayrton Shunt



(Multi-range ammeter using universal shunt)

For switch at a position 1

$$R_1 = \frac{R_m}{(m-1)}$$

For switch at a position 2

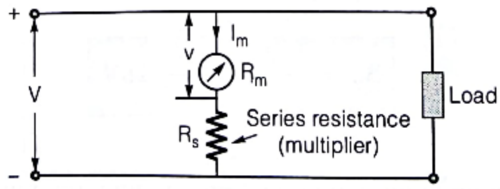
$$R_2 = \frac{(R_1 + R_m)}{m_2}$$

For switch at a position 3

$$R_3 = \frac{(R_1 + R_m)}{m_3}$$

Where, $m_1 = \frac{I_1}{I_m}, m_2 = \frac{I_2}{I_m}, m_3 = \frac{I_3}{I_m}$

3. Voltmeter Multipliers



Multiplying factor for multiplier

$$m = \frac{V}{v} = 1 + \frac{R_s}{R_m}$$

Resistance of multiplier

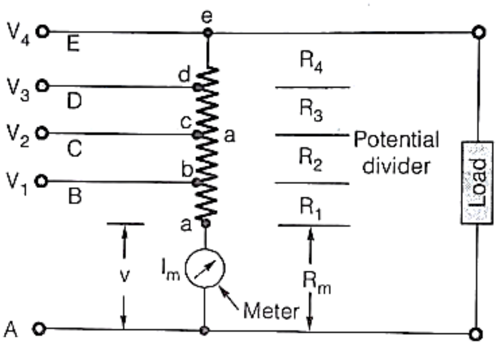
$$R_S = (m-1)R_m$$

Where,

R_s = Multiplier resistance

R_m = Internal resistance of meter

4. Potential Divider Arrangement



$R_1 = (m_1 - 1) R_m$

$R_2 = (m_2 - m_1) R_m$

$R_3 = (m_3 - m_2) R_m$

$R_4 = (m_4 - m_3) R_m$

Where,

R_1 = Resistance between point a and b

R_2 = Resistance between point b and c

R_3 = Resistance between point a and d

R_4 = Resistance between point d and e

Voltmeter Sensitivity (S_V)

$$S_V = \frac{1}{I_{fs}} = \frac{R_s + R_m}{V}\ \Omega/V$$

Remember : ....

To reduce loading effect, a voltmeter with higher value of sensitivity is preferred.

....

Moving Iron Instruments

Deflecting torque

$$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}$$

Deflection

$$\theta = \frac{1}{2}\frac{I^2}{K}\frac{dL}{d\theta}$$

For linear scale

$$\theta . \frac{dL}{d\theta} = \text{constant}$$

Scale is cramped at lower and higher end.

Note :..

- Moving iron instrument measure both A.C. and D.C. quantities.
- In case of A.C., it measure RMS value.
- Scale is non linear.
- Controlling torque is provided by spring and air friction damping is used.
- Curve between $\frac{dL}{d\theta}$ and θ is rectangular hyperbola.

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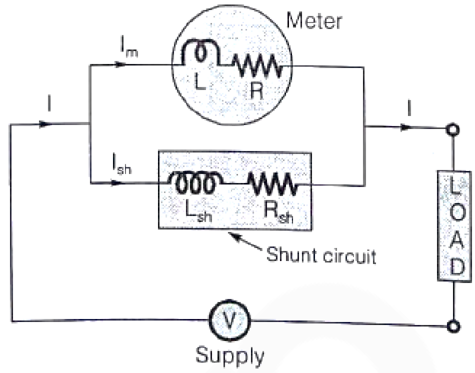
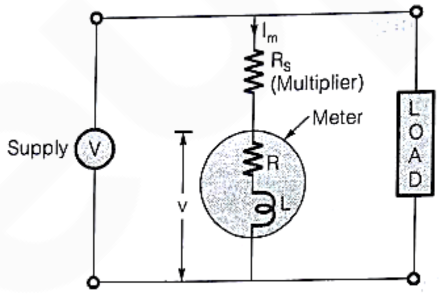
Shunts for Moving Iron Instruments



$$\frac{I_{sh}}{I_m} = \frac{R}{R_{sh}} \frac{\sqrt{1 + (\omega L / R)^2}}{\sqrt{1 + (\omega L_{sh} / R_{sh})^2}}$$

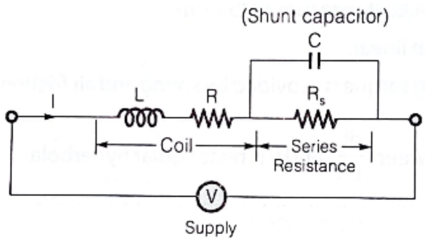
Multipliers for Moving Iron Instruments



Voltage multiplying factor

$$m = \frac{V}{V} = \frac{\sqrt{(R + R_s)^2 + \omega^2 L^2}}{\sqrt{R^2 + \omega^2 L^2}}$$

Errors in Moving Iron Instruments



Shunt capacitance

$$C = 0.41 \frac{L}{R_s^2}$$

Eddy currents

When ω is small

$$I_e' = \frac{\omega^2 M L_e I}{R_e^2}$$

When ω is large

$$I_e' = \frac{MI}{L_e} = \text{constant}$$

Where,

R_e, L_e = resistance and inductance of eddy current path

Note : ……………………………………………………………….

- Moving iron instrument is not suitable for measurement of current or voltage for frequency above 125 Hz because eddy current is constant at higher frequency.
- If meter time constant is equal to shunt time constant then ammeter is made independent of input supply frequency.
- The voltmeter is made independent of input supply frequency by connecting a capacitor in parallel to the series multiplier resistance R_S.
- To reduce hysteresis error, the iron part of moving iron is made up of Nickel iron alloy.
- To reduce the external stray magnetic field, the instrument is kept inside the iron case or iron shielding is done.

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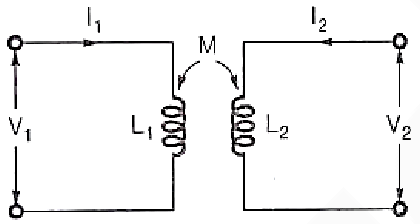
Electrodynamometer



(a) If i_1 and i_2 and D.C. current i.e. $i_1 = i_2 = I$

$$T_d = I^2 \frac{dM}{d\theta} \quad \text{(Measure average value)}$$

(b) If i_1 and i_2 are A.C. current and no phase shift

$i_1 = i_2 = I$

$$T_d = I^2 \frac{dM}{d\theta} \quad \text{(Measure RMS value)}$$

(c) If $i_1 = I_{m1} \sin \omega t$ and $i_2 = i_{m2} \sin(\omega t - \phi)$

$$T_d = I_1 I_2 \cos\phi \frac{dM}{d\theta} \quad \text{(Measure RMS value)}$$

Where, $I_1 = \frac{I_{m1}}{\sqrt{2}}$ and $I_2 = \frac{I_{m2}}{\sqrt{2}}$

Note : ……………………………………………………………….

- Electrodynamometer instrument is a transfer instrument.
- It measures both A.C. and D.C.
- Scale is nonlinear.
- Its sensitivity is lesser than PMMC and M.I. type instruments.

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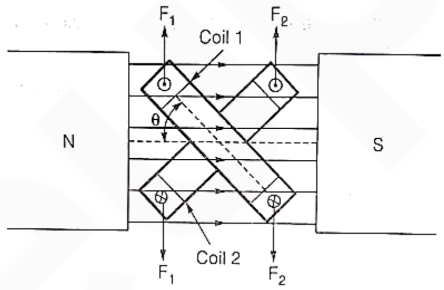
Ratiometer



Deflecting torque acting on coil 1

$$T_{d1} = N_1 B I_1 d_1 I_1 \cos\theta$$

Deflecting torque acting on coil 2

$$T_{d2} = N_2 B I_2 d_2 I_2 \cos\theta$$

Where,

I_1, I_2 = current in coil 1 and 2

N_1, N_2 = number of turns in coil 1 and 2

l_2, l_2 = length of coil 1 and 2

d_1, d_2 = width of coil 1 and 2

B = flux density of magnetic field

Deflection at equilibrium

$$\theta = k\left(\frac{I_1}{I_2}\right)$$

Chapter 5
Instrument Transformers

Ratios of Instrument Transformers

1. Transformation Ratio (R)

It is the ratio of the magnitude of the primary phasor to the secondary phasor.

$$R = \frac{|\text{primary phasor}|}{\|\text{secondary phasor}\|}$$

For current transformer (C.T.)

$$R = \frac{\text{primary winding current}}{\text{secondary winding current}}$$

For potential transformer (P.T.)

$$R = \frac{\text{primary winding voltage}}{\text{secondary winding voltage}}$$

2. Nominal Ratio (K_n)

It is the ratio of rated primary winding current (or voltage) to the rated secondary winding current (or voltage).

For C.T.

$$K_n = \frac{\text{rated primary winding current}}{\text{rated secondary winding current}}$$

For P.T.

$$K_n = \frac{\text{rated primary winding voltage}}{\text{rated secondary winding voltage}}$$

3. Turns Ratio (n)

For C.T.

$$n = \frac{\text{number of turns of secondary winding}}{\text{number of turns of primary winding}}$$

For P.T.

$$n = \frac{\text{number of turns of primary winding}}{\text{number of turns of secondary winding}}$$

4. Ratio Correction Factor

$$RCF = \frac{R}{K_n}$$

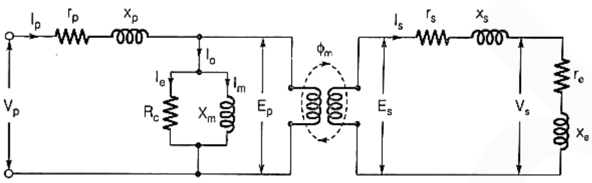
Remember : ……………………………………………………….

The ratio marked on the transformers is their nominal ratio.

……………………………………………………………………………

Current Transformer

Equivalent Circuit



Where,

r_s, x_s = resistance, reactance of secondary winding

r_e, x_e = resistance, reactance of external burden

E_p, E_s = primary and secondary winding induced voltage

N_p, N_s = number of primary and secondary winding turns

I_p, I_s = primary and secondary winding current

ϕ = primary and secondary winding current

θ = phase angle of transformer

δ = angle between secondary winding induced voltage and secondary winding current

Δ = phase angle of secondary winding load circuit

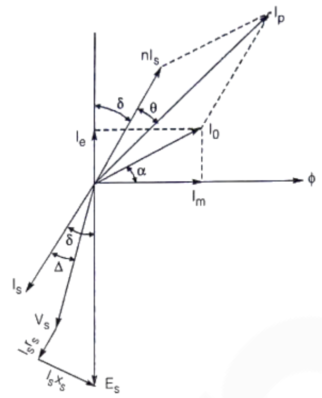
I_0 = exciting current

I_m = magnetizing component of exciting current

I_e = loss component of exciting current

α = angle between exciting current and flux

Phasor Diagram



Phasor diagram of C.T.

Transformation ratio

$$R = \frac{I_p}{I_s} n + \frac{I_o}{I_s}\sin(\delta + \alpha)$$

$$R \simeq \quad \frac{\sin\delta + I_e \cos\delta)}{I_s}$$

$$R \simeq \quad I_s \quad \simeq \quad I_p$$

Where, $I_m = I_o \cos \alpha$

$I_e = I_o \sin \alpha$

Phase angle

$$\theta \simeq \quad \left(\frac{{}_n \cos\delta - I_e \sin\delta}{nI_s} \right) \text{degree}$$

$$\theta \simeq \quad I_p \quad \text{degree}$$

Ratio error

$$\text{Ratio error} = \frac{\text{nominal ratio}(K_n) - \text{actual ratio}(R)}{\text{actual ratio}(R)}$$

Remember :……………………………………………………….

- The primary current of C.T. is depending on load connected to system but it is not depending secondary winding burden.
- Primary winding is single turn or bar winding and secondary has more number of turns to reduce the current at secondary.
- If primary current is very high, it causes reduction in ratio error and phase angle error. So to increase value of primary current the primary is maintain with single turn.
- The secondary number of turns are reduce by 1 or 2 turns then the ratio error reduces.

……………………………………………………………………………..

Potential transformer

Actual transformation (voltage) ratio

$$R = n + \frac{\frac{I_s}{n}\left[R_p \cos\Delta + X_p \sin\Delta\right] + I_e r_p + I_m x_p}{V_s}$$

Phase angle

$$\theta = \frac{I_s}{V_s}\left(X_s \cos\Delta - R_s \sin\Delta\right) + \frac{I_e x_p - I_m r_p}{nV_s} \text{rad.}$$

Note : ....

- C.T. never operates with secondary winding open but P.T. can be operated with secondary winding open.
- Strip wound core is used to reduce ratio error and phase angle error.

....

Application of C.T. and P.T.

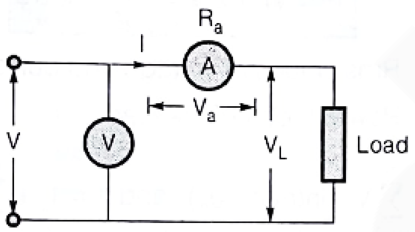
- Multiple operation with a single device.
- Higher current and higher voltages are step down to lower current and lower voltage so that metering is easier.
- Measuring circuit is isolated from the power circuit.
- Low power consumption.
- Replacement is easier.

Chapter 6
Measurement of Power and Wattmeters

Measurement of Power

1. D.C. Circuits

Ammeter connected between load and voltmeter



Power consumed by load :

$$P = VI - I^2 R_a$$

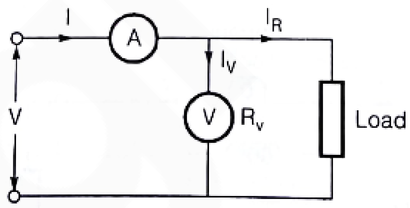
Where,

V = Voltage across voltmeter

I = Current through ammeter

R_a = Resistance of ammeter

Voltmeter connected between load and ammeter



Power consumed by load :

$$P = VI - \frac{V^2}{R_V}$$

Where,

V = Voltage across voltmeter

I = Current through ammeter

R_V = Resistance of voltmeter

2. A.C. Circuits

Instantaneous power

$$p = vi = V_m I_m \sin\omega t \cdot \sin(\omega t - \phi)$$

Where, $v = V_m \sin \omega t$

$i = I_m \sin(\omega t - \phi)$

Average power

$$P = VI\cos\phi = \frac{V_m I_m}{2}\cos\phi$$

Where,

V, I = Rms values of voltage and current

$\cos\phi$ = Power factor of the load

Let $v = V_o + \sum_{n=1}^{m} V_n \sin(n\omega t + \theta_n)$ and

$i = I_o + \sum_{n=1}^{m} I_n \sin(n\omega t + \phi_n)$ then

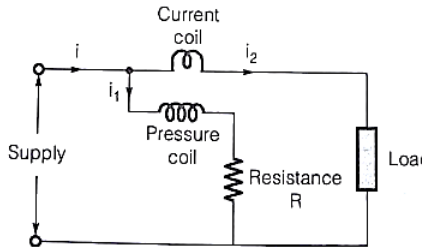
$$P_{avg.} = V_o I_o + \frac{1}{2}\sum_{n=1}^{m} V_n I_n \cos\left[\theta_n - \phi_n\right]$$

Remember : ....

Wattmeter reads average active power.

....

Electrodynamometer Wattmeter



Instantaneous torque

$$T_i = i_1 i_2 \left(\frac{dM}{d\theta}\right)$$

where, i_1, i_2 = instantaneous value of current in pressure and current coils

Deflecting torque

$$T_d = \frac{VI}{R_p}\cos\phi.\frac{dM}{d\theta}$$

Where,

R_p = resistance of pressure coil circuit

Controlling torque

$$T_c = K\theta$$

Where, K = spring constant

θ = final steady deflection

Deflection

$$\theta = \left(K_1\frac{dM}{d\theta}\right)P$$

Where,

P = power being measured = VI cos ϕ

$$K_1 = \frac{1}{R_p K}$$

Note : ……………………………………………………………………..

Scale is linear in terms of power as θ ∞ P.

……………………………………………………………………………………

Errors in Electrodynamometer Wattmeters

Correction fActor (K)

The correction factor is a factor by which the actual wattmeter reading is multiplied to get the true power.

For lagging power factor

$$K = \frac{\cos\phi}{\cos\beta\cos(\phi-\beta)}$$

For leading power factor

$$K = \frac{\cos\phi}{\cos\beta\cos(\phi+\beta)}$$

Where,

ϕ = Angle between current in the current coil and voltage of pressure coil

β = Angle between current and voltage of pressure coil

True power = Correction factor × actual wattmeter reading

For β very small

Actual wattmeter reading = true power (1 + tan ϕ tan β)

Error = tan ϕ tan β × true power = VI sin ϕ tan β

%error = tan ϕ tan β × 100

True power = VI cos ϕ

Where,

V = Voltage applied to pressure coil

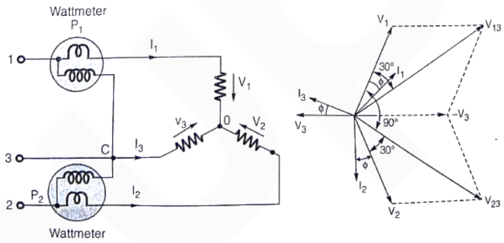
I = Current in current coil

Power in Poly-Phase Systems

Blondel's Theorem

If a network is supplied through n conductors, the total power is measured by summing the reading of n wattmeters so arranged that a current element of a wattmeter is in each line and the corresponding voltage element is connected between that line and a common point, if the common point is located on one of the lines, then the power may be measured by (n – 1) wattmeters.

Two wattmeter method



Reading of P1 wattmeter

$$P_1 = \sqrt{3}\,VI\cos(30^\circ - \phi)$$

Reading of P2 wattmeter

$$P_2 = \sqrt{3}\,VI\cos(30^\circ + \phi)$$

Total power consumed by load

$P = P_1 + P_2$

Power factor

$$\cos\phi = \cos\left[\tan^{-1}\left(\sqrt{3}\frac{P_1-P_2}{P_1+P_2}\right)\right]$$

$$\tan\phi = \sqrt{3}\frac{P_1-P_2}{P_1+P_2}$$

Where,

V = Phase voltage

I = Phase current

ϕ = Angle between phase current and phase voltage

Reading of wattmeter at Different Power Factor

S. No.	ϕ	cos ϕ	P_1	P_2	$P = P_1 + P_2$	Comment
1.	0	1	$\frac{\sqrt{3}}{2}V_LI_L$	$\frac{\sqrt{3}}{2}V_LI_L$	$\sqrt{3}\ V_LI_L$	P1 = P2 (equal reading)
2.	30°	0.866	V_LI_L	$\frac{V_LI_L}{2}$	$1.5\ V_LI_L$	$P_1 = 2P_2$
3.	60°	0.5	$\frac{\sqrt{3}}{2}V_LI_L$	0	$\frac{\sqrt{3}}{2}V_LI_L$	$P_2 = 0$, $P_2 = P$
4.	90°	0	$+\frac{V_LI_L}{2}$	$-\frac{V_LI_L}{2}$	0	$P_1 = -P_2$

Note : ....

When wattmeter reading comes into negative, reverse either P.C. or C.C. terminal and then take the reading of negative wattmeter.

....

Measurement of Energy

For the measurement of energy, we use energy meter. Energy meter is an integrating instrument which adds the energy cumulatively over a period of time.

$$\text{Energy} = \text{Power} \times \text{time}$$

$$\text{Energy} = \int_0^t P \cdot dt \text{ kWhr}$$

Note : ....

- Energy meter works on principle of induction motor.
- The meter which measure A.C. energy is called watt hour meter.
- The meter which measure D.C. energy is called amp-hour meter.

....

Deflection torque

$$T_D \propto P$$

Breaking torque

$$T_B \propto N$$

Where, N = Speed of disc in rps

At balance

$$T_d = T_B$$

$$\int P \cdot dt = K \int N \cdot dt$$

$$\text{Energy} \propto \int N \cdot dt$$

Energy meter constant (EMC)

$$\text{EMC} = \frac{\text{Number of revolution made by disc}}{\text{Energy recorded in kWhr.}}$$

Where, P = Power in kW

t = Time in hrs.

$$\%\,\text{Creeping Error} = \frac{\begin{matrix}\text{Revolution of disc due}\\ \text{to creeping per hour}\end{matrix}}{\begin{matrix}\text{Revolution of disc due}\\ \text{to total load per hour}\end{matrix}} \times 100$$

Remember : ....

Potential coil of energy meter should be highly inductive so that it measures true energy.

....`

Compensation in Energy Meter

1. Lag compensation : Through lag coil or shading coil.
2. Low load or friction adjustment : By using shading loop.
3. Over friction or creeping : By providing holes or slots on rotating disc.
4. Over load compensation : By keeping saturable shunt magnet in series magnet or current coil.
5. Over voltage compensation : By keeping saturable shunt magnet in shunt magnet.
6. Temperature compensation : By making permanent magnet of "mutemp" material.
7. Speed adjustment : By adjusting position of break magnet.

Remember : ....

Creeping error is always positive.

If either potential coil or current coil is wrongly connected then the disc rotates in opposite direction.

....

Chapter 7

Measurement of Resistance

Classification of Resistance

1. Low resistance : All resistance of the order of 1 Ω and below.

   Example : Winding coils of electrical motors, generators and transformers.
2. Medium resistance : Resistances form 1 Ω upwards to about 0.1 MΩ.

   Example : Resistance of heaters, potentiometers.
3. High resistance : All resistances of the order of 0.1 MΩ and above.

   Example : Insulation of electrical cable and windings, insulation of motors, generators and transformers.

Measurement of Medium Resistance

The different methods employed are :

(i) Ammeter – voltmeter method

(ii) Wheatstone bridge method

(iii) Ohmmeter method

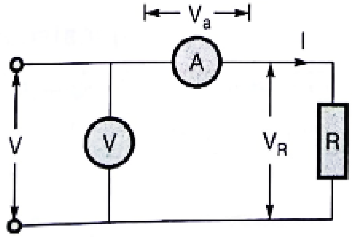
(iv) Substitution method

1. **Ammeter Voltmeter Method**

$$R_m = \frac{\text{voltmeter reading}}{\text{ammeter reading}} = \frac{V}{I}$$

Where, R_m = measured value of resistance

(a) Circuit for higher resistance



True value of resistance

$$R = R_{m_1} - R_a$$

$$R_m = R_{m_1}\left(1 - \frac{R_a}{R_{m_1}}\right)$$

Where,

R_{m_1} = = Measured value of resistance

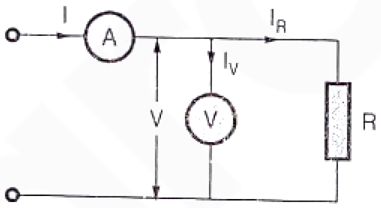
R_a = Resistance of ammeter

Relative error

$$\epsilon_r = \frac{R_{m_1} - R}{R} = \frac{R_a}{R}$$

To get minimum error, the test resistance should be more than the ammeter resistance so that this adjustment is suitable for measurement of high resistance.

(b) Circuit for lower resistance



True value of resistance

$$R = \frac{R_{m_2} R_v}{R_v - R_{m_2}}$$

Where,

R_{m_2} = Measured value of resistance

R_v = Resistance of voltmeter

For $R_v >> R_{m_2}$

$$R = R_{m_2}\left(1 + \frac{R_{m_2}}{R_v}\right)$$

Relative error

$$\epsilon_r = \frac{R_{m_2} - R}{R} = \frac{-R^2_{m2}}{R_v R} \qquad [R_v >> R_{m_2}]$$

Approximate relative error

$$\epsilon = -\frac{R}{R_v} \qquad \left[\text{For } R_{m_2} \approx R\right]$$

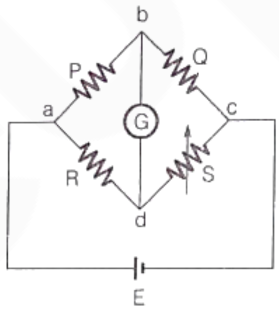
- This circuit is suitable for measurement of low resistance under medium scale range to get minimum error.

Note : ....

Relative errors for above two cases are equal when true value of resistance, $R = \sqrt{R_a R_v}$

....

2. **Wheatstone Bridge**



At balance

$$R = S\frac{P}{Q}$$

Sensitivity of Wheatstone bridge

$$S_B = \frac{\theta}{\Delta R / R} = \frac{S_v ESR}{(R+S)^2}; \text{mm}$$

$$S_B = \frac{S_v E}{\frac{P}{Q} + 2 + \frac{Q}{P}}$$

Where,

S_v = Voltage sensitivity of galvanometer, mm/volt

E = Bridge voltage

P, Q = Branch resistances

θ = Deflection of galvanometer, mm

For a bridge with equal arms

$$S_B = \frac{S_v E}{4}$$

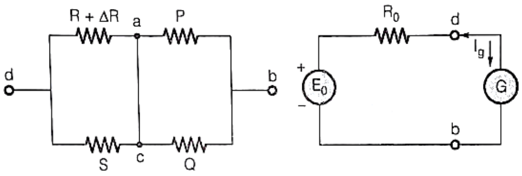
Note : ....

- For maximum bridge sensitivity

$$\frac{P}{Q} = \frac{R}{S} = 1$$

- Sensitivity of bridge is most important parameter as compared to accuracy, precision and resolution.

Equivalent circuit of Wheatstone bridge



Galvanometer current

$$I_g = \frac{E_0}{R_0 + G}$$

Where

E_0 = Thenvenin's or open circuit voltage appearing between terminals b and d with galvanometer circuit open circuited.

G = Resistance of the galvanometer circuit

$$E_0 = E\left[\frac{R+\Delta R}{2R+\Delta R} - \frac{1}{2}\right] \approx E\left(\frac{\Delta R}{4R}\right) \text{ as DR << R}$$

ΔR = Change in resistance R

Thevenin equivalent resistance of bridge

$$R_0 = \frac{RS}{R+S} + \frac{PQ}{P+Q}$$

Galvanometer deflection

$$\theta = \frac{S_V ES\,\Delta R}{(R+S)^2} = \frac{S_i\,ES\Delta R}{(R_0+G)(R+S)^2}$$

Where,

S_i = Current sensitivity of galvanometer

Bridge sensitivity

$$S_B = \frac{\theta}{\Delta R / R} = \frac{S_i\,E\,SR}{(R_0+G)(R+S)^2}$$

Current sensitivity

$$S_i = \frac{\theta}{I_g};\text{mm/μA}$$

θ = Deflection in galvanometer

I_g = Current in galvanometer

Voltage Sensitivity

$$S_v = \frac{\theta}{V_{Th}};\text{mm/V}$$

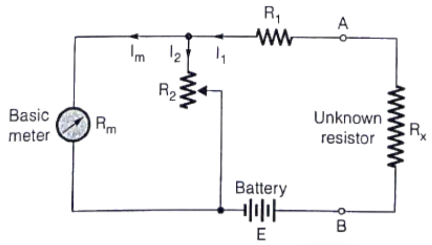
V_{Th} = Voltage across galvanometer

Note :

In Wheatstone bridge method, the effect of lead resistance is not eliminated hence it is not suitable for measurement of low resistance.

3. Ohmmeters

(a) Series Type Ohmmeter



Half scale resistance

$$R_h = R_1 + \frac{R_2 R_m}{R_2 + R_m}$$

Meter current

$$I_m = \frac{ER_2}{(R_h + R_x)(R_2 + R_m)}$$

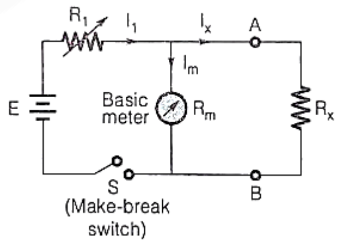
Full scale deflection current

$$I_{fs} = \frac{ER^2}{R_h(R_2 + R_m)}$$

Friction of full scale reading

$$S = \frac{I_m}{I_{fs}} = \frac{R_m}{R_x + R_h}$$

(b) Shunt Type Ohmmeter



Half scale reading of unknown resistance R_X is

$$R_h = \frac{R_1 R_m}{R_1 + R_m}$$

Half scale reading of the meter

$$I_h = 0.5 I_{fs} = \frac{ER_h}{R_1 R_m + R_h(R_1 + R_m)}$$

Where,

R_m = Internal resistance of meter

R_1 = Adjustable resistor (as shown in figure)

E = Supply voltage

Measurement of low Resistance

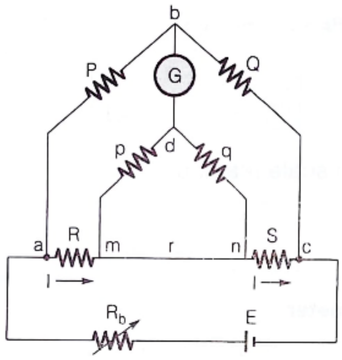
The different methods employed are :

(i) Kelvin's double bridge method

(ii) Ammeter voltmeter method

(iii) Potentiometer method

Kelvin's Double Bridge Method



For zero galvanometer deflection

$$E_{ab} = E_{amd}$$

$$R = \frac{P}{Q}.S + \frac{qr}{p+q+r}\left[\frac{P}{Q} - \frac{p}{q}\right]$$

If $\frac{P}{Q} = \frac{p}{q}$

then $R = \frac{P}{Q}.S$

Note : ....

Accuracies by Kelvin double bridge method

(i) From 1000 μΩ to 1.0 Ω : 0.005%

(ii) From 100 μΩ to 1000 μΩ : 0.5% to 0.05%

(iii) From 10 μΩ to 100 μΩ : 0.5% to 0.2%

....

Measurement of High Resistance

The difference methods employed are :

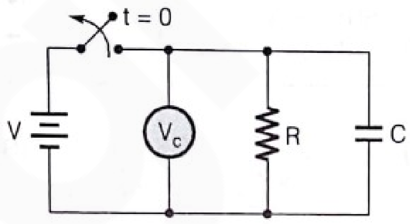
(i) Loss of charge method

(ii) Meggar

(iii) Direct deflection method

(iv) Megohm bridge

1. Loss of Charge Method



$$R = \frac{0.4343t}{C\log_{10}\left(\frac{V}{V_C}\right)}$$

2. Meggar

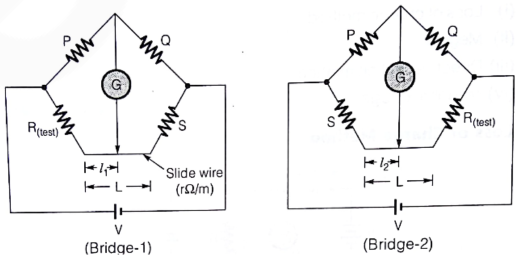
- Meggar works on the principle of electrodynamometer.
- Meggar is used to measure the insulation resistance of cable, motor and generator, etc.
- Deflecting torque angle is proportional to the resistance of the insulator, which is under test.
- It is also used to check the continuity of cable.
- No external control torque provided.
- Air friction damping is used.
- No need of external power supply.

Note :..

High resistance have a guard terminal which is used to avoid leakage current.

....

Carry Foster Slide Wire Bridge



From bridge (1)

$$\frac{P}{Q} + 1 = \frac{R+S+Lr}{S+(L-l_1)r} \quad ...(i)$$

From bridge (2)

$$\frac{P}{Q} + 1 = \frac{R+S+Lr}{R+(L-l_2)r} \quad ...(ii)$$

Equating equation (i) and (ii)

$$R - S = (l_2 - l_1)r$$

Note : ....

Carry Foster bridge method is used for medium resistance measurement by comparing with standard resistance.

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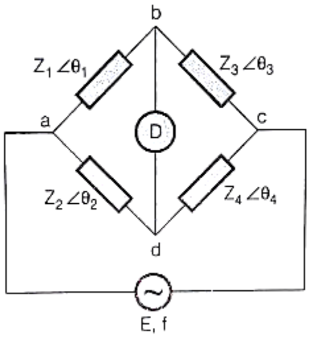
Chapter 8
A.C. Bridges

Introduction

Used to measure self inductance, Mutual inductance, capacitance, and frequency.



General equation for bridge balance

$$\bar{Z}_1 \bar{Z}_4 = \bar{Z}_2 \bar{Z}_3$$

Magnitude condition

$$|Z_1||Z_4| = |Z_2||Z_3|$$

Angle condition

$$\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$$

Note :……………………………………………………………………..

Magnitude condition and angle condition both must be satisfied for the bridge to be balanced.

……………………………………………………………………………….

Depending upon the frequency, different null detectors are used

Vibration galvanometer - 5 Hz to 1 kHz

Head phones - 250 Hz to 4kHz

Tuned amplifier detector - 10Hz to 100kHz

D'Arsonval Galvanometer-DC frequency=0 Hz

Depending upon phase angle θ, elements are

θ	Elements
0^o	R
90^o	L_1
-90^o	C_1
$0^o < \theta_1 < 90^o$	R_1, L_1
$-90 < \theta_1 < 0^o$	R_1, C_1

Convergence to balance point :

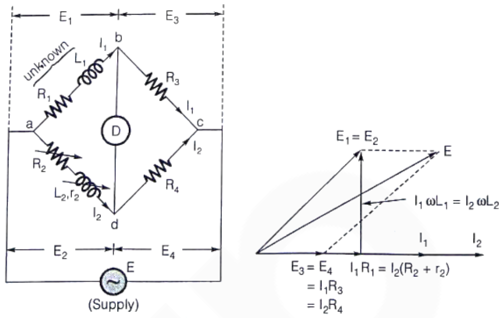
If the variables are in the same arm of bridge then minimum time is required for balancing of bridge. This is called convergence to balance point.

Quality factor (Q.F.)

$$Q.F. = \frac{\text{Energy stored}}{\text{Energy disspiated}}$$

Measurement of Self Inductance

1. Maxwell's Inductance Bridge



$$R_1 = \frac{R_3}{R_4}(R_2 + r_2)$$

$$L_1 = \frac{R_3}{R_4}.L_4$$

Where,

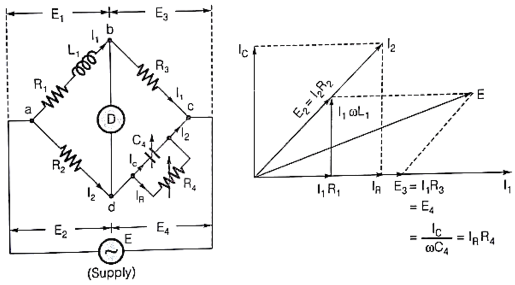
L_1 = Unknown inductance of resistance R_1

L_2 = Variable inductance of fixed resistance r_2

R_2 = Variable resistance connected in series with L_2

R_3, R_4 = Known non-inductive resistance

2. Maxwell's Inductance-Capacitance Bridge



$$R_1 = \frac{R_2R_3}{R_4} \text{ and } L_1 = R_2R_3C_4$$

Q factor of the coil

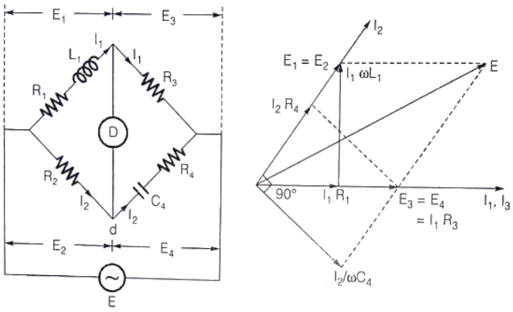
$$Q = \frac{\omega L_1}{R_1} = \omega C_4 R_4$$

Note : ……………………………………………………………………

- Not suitable for measurement of high Q coil because phase angle criteria does not satisfy.
- Not suitable for measurement of low Q-coil because of sliding balance problem.
- Suitable for measurement of medium Q coil i.e. (1 < Q < 10).

……………………………………………………………………………….

3. Hay's Bridge



$$R_1 = \frac{\omega^2 R_2 R_3 R_4 C_4^2}{1+\omega^2 C_4^2 R_4^2} = \frac{R_2 R_3}{R_4}\left(\frac{1}{1+Q^2}\right)$$

$$L_1 = \frac{R_2 R_3 C_4}{1+\omega^2 C_4^2 R_4^2} = \frac{R_2 R_3 C_4}{1+\left(\frac{1}{Q}\right)^2}$$

For Q > 10

$$L_1 = R_2 R_3 C_4$$

$$Q = \frac{\omega L_1}{R_1} = \frac{1}{\omega C_4 R_4}$$

Where,

L_1 = Unknown-inductance having a resistance R1

R_2, R_3, R_4 = known non-inductive resistance

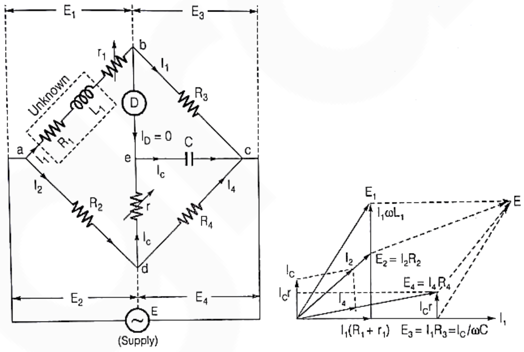
C_4 = Standard capacitor

Note : ……………………………………………………………………..

The Hay's bridge is suited for the measurement of high Q inductors.

……………………………………………………………………………………..

4. Anderson's Bridge



$$R_1 = \frac{R_2 R_3}{R_4} - r_1$$

$$L_1 = \frac{CR_3}{R_4}\left[r\left(R_4+R_2\right)+R_2 R_4\right]$$

Where,

L_1 = Self-inductance to be measured

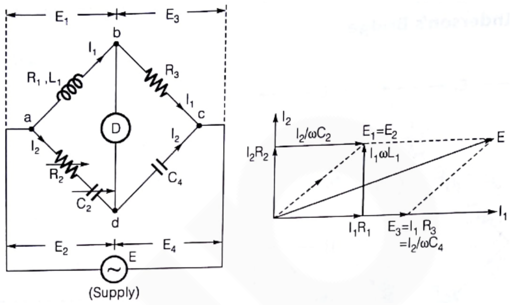
R_1 = Resistance of self-inductor

r_1 = Resistance connected in series with self-inductor

r, R_2, R_3, R_4 = Known non-inductive resistances

C = Fixed standard capacitor

5. Owen's Bridge



$$L_1 = R_2 R_3 C_4$$

$$R_1 = R_3 \frac{C_4}{C_2}$$

Where,

L_1 = Unknown self inductance of resistance R_1

R_2 = Variable non-inductive resistance

R_3 = Fixed non-inductive resistance

C_2 = Variable standard capacitor

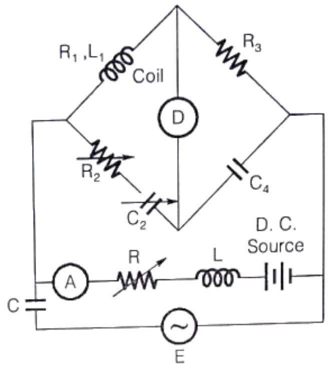
C_4 = Fixed standard capacitor

Note : ……………………………………………………………………

Owen's bridge is used for measurement of unknown inductance and incremental inductance and incremental permeability (μ).

……………………………………………………………………………

Measurement of Incremental Inductance



Incremental inductance

$$L_1 = R_2 R_3 C_4$$

Incremental permeability

$$\mu = \frac{L_1 l}{N^2 A}$$

Where,

N = Number of turns

A = Area of flux path
l = Length of flux path
R_2 = Variable non-inductive resistance
R_3 = Fixed non-inductive resistance
C_4 = Fixed standard capacitor

Note : ....

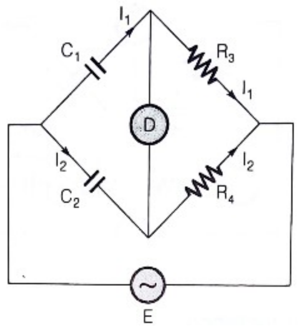
- External D.C. source is used to compensate residual magnetism.
- Capacitor, C is to block D.C. to enter into A.C. and inductor, L is to block A.C. to enter into D.C.

....

Measurement of Capacitance

1. De Sauty's Bridge

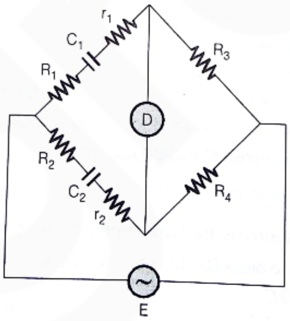
(a) For lossless capacitor



$$C_1 = \frac{C_2 R_4}{R_4}$$

Where, C_1 = Capacitor whose Capacitance to be measured
C_2 = A standard capacitor
R_3, R_4 = Non-inductive resistors

(b) For imperfect capacitor having dielectric loss



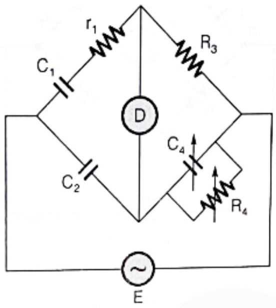
$$\frac{C_1}{C_2} = \frac{R_4}{R_3} = \frac{R_2 + r_2}{R_1 + r_1}$$

Where,
r_1, r_2 = Resistance representing the loss component of the two capacitor.

Dissipation factor

$$D = \tan\delta = \omega C_1 r_1 = \omega C_2 r_2$$

2. Schering Bridge



$$r_1 = \frac{R_3 C_4}{C_2}$$

$$C_1 = C_2\left(\frac{R_4}{R_3}\right)$$

Dissipation factor

$$D = \omega C_1 r_1 = \omega C_4 R_4$$

where,
C_1 = capacitor whose capacitance is to be determined
r_1 = Series resistance representing the loss in the capacitor C_1
C_2 = Standard loss-free capacitor
R_3 = Non-inductive resistance
C_4 = Variable capacitor
R_4 = Variable non-inductive resistance in parallel with variable capacitor C_4

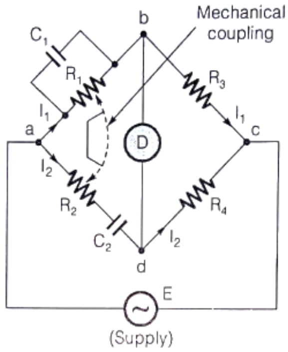
Note : ....

Schering bridge is shielded with metal screen to reduce the stray capacitance exists between the arms and arms to the earth.

....

Measurement of Frequency

Wien's Brdige



Frequency

$$\frac{R_4}{R_3} = \frac{R_2}{R_1} + \frac{C_1}{C_2}$$

$$f = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} \text{Hz}$$

For $R_1 = R_2 = R$ and $C_1 = C_2 = C$

$$f = \frac{1}{2\pi RC}$$

$$\frac{R_4}{R_3} = 2$$

Limitation of Wein's Bridge

If the input signal is not a sinusoidal or signal containing harmonics then balancing of bridge is not possible because null detector is sensitive to the frequencies.

Chapter 9
Magnetic Measurements

Flux density

$$B = \frac{\phi}{A_s} = \frac{R K_q \theta_1}{2NA_s}$$

where,

ϕ = Flux linking search coil

A_s = Cross-sectional area of specimen

R = Resistance of the ballistic galvanometer circuit

$K_q \ \theta_1$ = Charge indicated by ballistic galvanometer

N = Number of turns in the search coil

Hysteresis loss per unit volume

$$p_h = \eta f\ B_m^k$$

where,

η = Hysteresis coefficient

f = Frequency; Hz

B_m = Maximum flux density; Wb/m^2

k = Steinmetz coefficient

Note : ………………………………………………………………….

The value of k varies from 1.6 to 2.

…………………………………………………………………………………

Eddy current loss per unit volume for laminations

$$p_e = \frac{4k_f^2 f^2 B_m^2 t^2}{2\rho}$$

Where, kf = Form factor

t = Thickness of laminations; m

ρ = Resistivity of material; Ω-m

Total iron loss per unit volume

$$p_i = p_h + p_e = \eta f B_m^k + \frac{4k_f^2 f^2 B_m^2 t^2}{3\rho}$$

Maximum flux density

$$B_m = \frac{E_2}{4k_f f A_s N_2}$$

For sinusoidal supply

$$B_m = \frac{E_2}{4.44 f A_s N_2}$$

Where,

E_2 = Voltage induced in secondary winding

$E_2 = 4kf\ f\ \phi_m\ N_2$

K_f = Form factor (= 1.11 for sinusoidal supply)

f = Frequency

A_s = Cross-sectional area of specimen

N_2 = Number of turns in secondary winding

ϕ_m = Maximum flux linkage

Chapter 10
Electronic Instruments

Average current through diode vacuum tube voltmeters

$$I_{av} = \frac{E_{av}}{2R} = \frac{E_{rms}}{2 \times 1.11 \times R} = 0.45\frac{E_{rms}}{R}$$

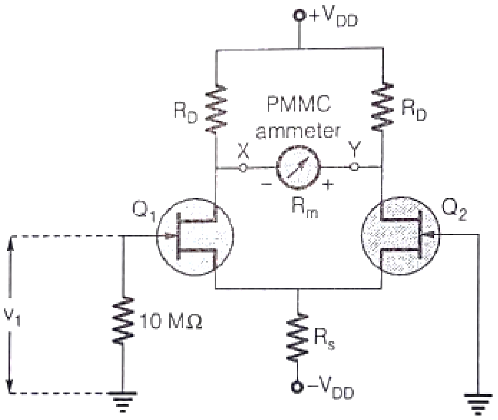
Where,

E_{rms} = RMS value of applied voltage

E_{av} = Average value of applied voltage

R = Load resistance

Difference amplifier type of electronic voltmeter



Thevenin's voltage across terminal X-Y

$$V_{Th} = g_m \left(\frac{r_d R_D}{r_d + R_D} \right) v_1$$

Thevenin's resistance looking into terminals X-Y

$$R_{Th} = \frac{2 r_d R_D}{r_d + R_D}$$

Where,

R_d = A.C. drain resistance in Ω

g_m = Transconductance in mho

Current through ammeter

$$i = \frac{V_{Th}}{R_{Th} + R_m} = \frac{g_m r_d R_D / (r_d + R_D)}{2r_d R_D / (r_d + R_D) + R_m} . V_1$$

Where, RD << rd

$$i = \frac{g_m R_D}{2R_D + R_m} . V_1$$

Digital Meters

Basic measurable quantity in digital meter is D.C.

1. **Resolution (R) of Digital Meter**
The smallest change in the input which a digital meter can be able to detect is called resolution.

$$R = \frac{1}{10^n}$$

where, n = Number of full digit.

2. **Sensitivity (S)**
The smallest change in input that can be displayed within given range.
S = Resolution × Range of meter

3. **Over ranging**
Switch on the extra half (1/2) is called over ranging. Due to this over ranging the range of the instrument increases.

Chapter 11
Cathode Ray Oscilloscope

CRO is a digital instrument, which works on the principle of thermionic emission i.e. emission of electron from a heated surface. It is a linear device. With the use of CRO one can measure peak to peak, rms, peak or average value of voltage and current.

Calibration of CRO

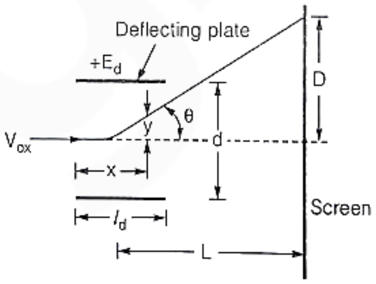
Calibration of CRO is done by applying a known quality of square signal having a frequency of 1 kHz and peak to peak magnitude of 1 mV.

The rise time (t_r), of signal applied to CRO and bandwidth of CRO are related as

tr × B.W. = 0.35

If this condition fails then the signal is distorted at the output of CRO.

Electrostatic Deflection



$$y = \frac{1}{2} \frac{e E_y}{m v_{ox}^2} x^2$$

where,

y = Displacement in y-direction; m

e = Charge of an electron; Coulomb

E_y = Electric field intensity in Y-direction; V/m

m = Mass of electron; kg

V_{ox} = Velocity of electron when entering the fields of deflecting plates; m/s

x = Displacement in x-direction; m

Deflection

$$D = \frac{L\ l_d E_d}{2d\ E_a}$$

where,

L = Distance between screen and the centre of deflecting plates; m

l_d = Length of deflecting plates; m

E_d = Potential between deflecting plates; V

d = Distance between deflecting plates; m

E_a = Voltage of pre-accelerating anode; V

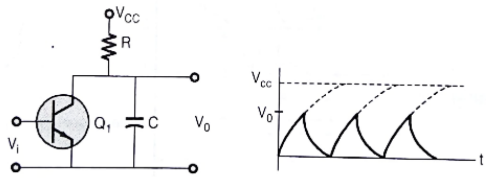
Deflection sensitivity

$$S = \frac{D}{E_d} = \frac{L l_d}{2d E_a} \text{ m/V}$$

Deflection factor

$$G = \frac{1}{S} = \frac{2d E_a}{L l_d} \text{ V/m}$$

Sawtooth Generator



$$V_0 = V_{cc}\left[1 - \exp\left(-t / RC\right)\right]$$

where,

V_0 = Instantaneous voltage across the capacitor at time t; V

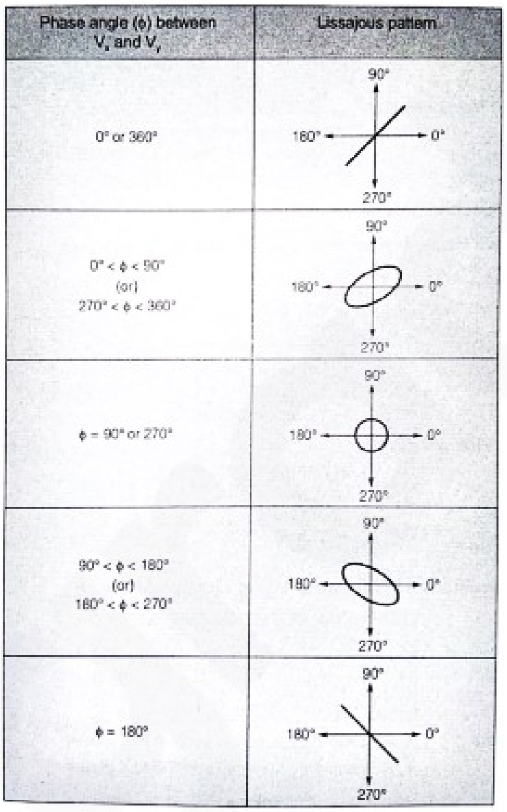
V_{cc} = Supply voltage

Lissajous patterns

If horizontal and vertical deflecting plate are applied with sinusoidal signal, the waveform pattern appearing on the screen is called Lissajous pattern.

Application

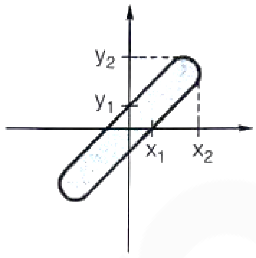
- Used for finding the phase angle difference between the signal applied to vertical and horizontal plate.
- Used for finding the frequency ratio between vertical and horizontal plates voltage.



Phase angle (ϕ) between V_x and V_y	Lissajous pattern
0° or 360°	90°, 180°, 0°, 270°
0° < ϕ < 90° (or) 270° < ϕ < 360°	90°, 180°, 0°, 270°
ϕ = 90° or 270°	90°, 180°, 0°, 270°
90° < ϕ < 180° (or) 180° < ϕ < 270°	90°, 180°, 0°, 270°
ϕ = 180°	90°, 180°, 0°, 270°

Finding the phase angle ϕ from given Lissajous pattern

(a) When Lissajous pattern is in first and third quadrant

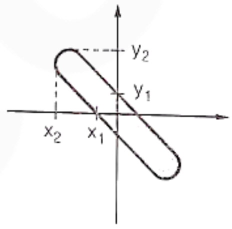


First possibility

$$\phi = \sin^{-1}\left(\frac{x_1}{x_2}\right) = \sin^{-1}\left(\frac{y_1}{y_2}\right)$$

Second possibility = 360° – ϕ

(b) When Lissajous pattern is in second and fourth quadrant



First possibility

$$\phi = 180° - \sin^{-1}\left(\frac{x_1}{x_2}\right)$$

Second possibility = 360° – ϕ

Measurement of Frequency Using Lissajous Pattern

$$\frac{f_y}{f_x} = \frac{\text{(number of intersections of the horizontal line with the curve)}}{\text{(number of intersections of the vertical line with the curve)}}$$

Where,

fy = Frequency of signal applied to Y plates

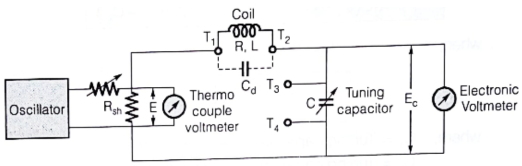
fx = Frequency of signal applied to X plates

Chapter 12
High Frequency Measurements

Q-meter

It works on the principle of series resonance.

Measurement of the Storage Factor Q



Resonant frequency of Q-Meter

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Measured value of Q

$$Q_m = \frac{\omega_0 L}{R + R_{sh}}$$

True value of Q

$$Q_t = \frac{\omega_0 L}{R}$$

$$Q_t = Q_m\left(1 + \frac{R_{sh}}{R}\right) = Q_m\left(1 + \frac{C_d}{C}\right)$$

Where,

R = Resistance of coil

L = Inductance of coil

R_{sh} = Shunt resistance

C = Tuning capacitance

C_d = Distributed or self-capacitance

Measurement of inductance

$$L = \frac{1}{4\pi^2 f_o^2 C}$$

Measurement of effective resistance

$$R = \frac{\omega_o L}{Q_t}$$

Measurement of Distributed or self-capacitance

Resonance frequency

$$f_1 = \frac{1}{2\pi\sqrt{L(C_1 + C_d)}};$$

$$f_2 = \frac{1}{2\pi\sqrt{L(C_2 + C_d)}}$$

When, $f_2 = nf_1$

then, $C_d = \frac{C_1 - n^2 C_2}{n^2 - 1}$

where,

C_1 = Tuning capacitance at frequency f_1

C_2 = Tuning capacitance at frequency f_2

Measurement of Unkown Capacitance C_x

Adjust capacitor C = C_1 to get resonance frequency f_1 with unknown capacitance C_x in parallel.

$$f_1 = \frac{1}{2\pi\sqrt{L(C_x + C_1)}} \quad ...(i)$$

Now remove C_x and again adjust C = C_2 to get same resonance frequency f_1

$$f_1 = \frac{1}{2\pi\sqrt{LC_2}} \quad ...(ii)$$

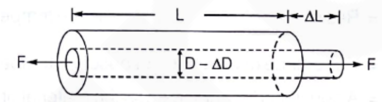
By equating equation (i) and (ii),

$$C_x = C_2 - C_1$$

Chapter 13

Transducers

Strain Gauge



Gauge factor of strain gauge

$$G_f = \frac{\Delta R / R}{\Delta L / L} = 1 + 2v + \frac{\Delta\rho / \rho}{\in}$$

Where,

$\frac{\Delta\rho}{\rho}$ = Per unit change in resistivity

v = Poission's ratio

$\in$ = Strain

For $\frac{\Delta\rho}{\rho} \to 0$

$$G_f \approx 1 + 2v$$

Poission's ratio

$$\text{Poisson's ratio}(v) = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$$= \frac{-\partial D / D}{\partial L / L}$$

Strain

$$\text{Strain}(\in) = \frac{\Delta L}{L}$$

where,

$\frac{\Delta L}{L}$ = Per unit change in length

Thermistor

Resistance of thermistor

$$R_{T_1} = R_{T_2} \exp\left[\beta\left(\frac{1}{T_2} - \frac{1}{T_2}\right)\right]$$

Where,

R_{T_1} = Resistance of thermistor at absolute temperature T_1 ; °K

R_{T_2} = Resistance of thermistor at absolute temperature T_2 ; °K

β = A constant depending upon the material of thermistor

Steinhart-Hart equation

$$\frac{1}{T} = A + B\ln R + C(\ln R)^3$$

where,

T = Temperature; °K,

R = Resistance of thermistor; Ω

A, B, C = Curve fitting constant

Thermistor resistance

$$R_T = aR_0 \exp(b/T)$$

Where,

R_T, R_0 = Resistance of thermistor at temperature T °K and ice point respectively

Thermocouple

E.M.F. produced in a thermocouple

$$E = a(\Delta\theta) + b(\Delta\theta)^2$$

Where

Δθ = Difference in temperature between the hot thermocouple junction and the reference junction of thermocouple; °C

a, b = Constant

LVDT

Sensitivity of LVDT

$$\text{Sensitivity} = \frac{\text{Output voltage}}{\text{displacement}}$$

Capacitive Transducers

Capacitance

Capacitance of parallel plate capacitor

$$C = \frac{\epsilon A}{d} = \frac{\epsilon xw}{d}$$

Where,

A = Overlapping area of plates

x = Length of overlapping part of plates

w = Width of overlapping part of plates

d = Distance between two plates

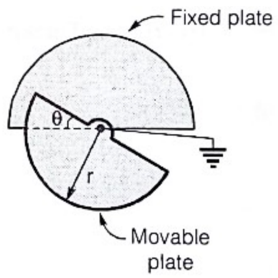
∈ = Permittivity of medium

Capacitance of cylindrical capacitor

$$C = \frac{2\pi \epsilon x}{\log_e(D_2/D_1)} F$$

Capacitance at angular displacement θ

$$C = \frac{\epsilon \theta r^2}{2d}$$



Where,

r = Radius of semi circular palte

θ = Angular displacement in radian

Sensitivity

Sensitivity of parallel plate capacitive transducer

$$S = \frac{\partial C}{\partial x} = \epsilon \frac{w}{d}$$

Where,

x = Length of overlapping part of cylinders; m

D_2 = Inner diameter of outer cylindrical electrode; m

D_1 = Outer diameter of inner cylindrical electrodes; m

Sensitivity of cylindrical capacitive transducer

$$S = \frac{2\pi \epsilon}{\log_e(D_2/D_1)} F/m$$

Sensitivity of variable capacitance transducer

$$S = \frac{\epsilon r^2}{2d}$$

Piezo-Electric Transducer

Voltage sensitivity of crystal

$$g = \frac{\text{Electric field}}{\text{Stress}} = \frac{\epsilon}{P} \text{Vm/N}$$

Where,

P = Pressure or stress; N/m^2

Charge sensitivity

$$d = \epsilon_r \epsilon_0 g \text{ C/N}$$

Output voltage

$$E_o = gtp$$

Where,

t = Thickness of crystal; m